An Overview of Linear Systems

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- Description

  This course provides an introduction to linear systems with a view towards modeling, simulation, filtering, and control system design. The material introduces linear, time-invariant systems that can be modeled with ordinary, constant coefficient, differential equations. The Laplace transform and transfer functions are used to simply the analysis. Bode and Nyquist plots are used to present the system frequency response.

- Module List:
  1) Modeling of continuous time invariant linear systems
  2) Analysis of linear systems, state space representation, numerical simulation
  3) Analysis of linear systems, control system design and synthesis
  4) Implementation of control systems, discretization, z-transforms.

- Author: Duane Mattern

- Background:

  Duane Mattern is an independent contractor specializing in modeling, simulation, control system design and implementation. He is experienced with rapid prototyping software tools like the Mathwork's Matlab/Simulink/Controls/RTW and Integrated System's Xmath/MatrixX/ SystemBuild/Autocode. As a mechanical engineer specializing in instrumentation and controls with more than 10 years of experience, he has a broad range of practical knowledge, including automatic testing machines, turbofan engine control, integrated flight and propulsion control, servo-systems including voice coil and electromagnetic actuation, diagnostics, and neural networks. His current interests are in modeling, real-time simulation, control system design and embedded system programming for control system implementation,

- Prerequisites:

  Familiarity with the following concepts: (i) phasor notation and the fundamentals of complex variables; (ii) integration and differentiation; (iii) superposition, the Laplace transform and transfer functions; (iv) frequency response using Bode and Nyquist plots, (v) basic linear algebra.

- Intended Audience:

  (1) Engineer or practitioner who would like to renew their knowledge of linear systems;
  (2) Engineer or practitioners who would like a fast introduction to linear systems;
  (3) College student who desires an alternative presentation to linear systems, separate from what they receive in their normal courses.

Estimated Total Learning Time:

2 hours

Module 9 Analyzing the control design

- Purpose
  - To introduce various linear system analysis techniques.
  - To demonstrate these techniques on a practical example.

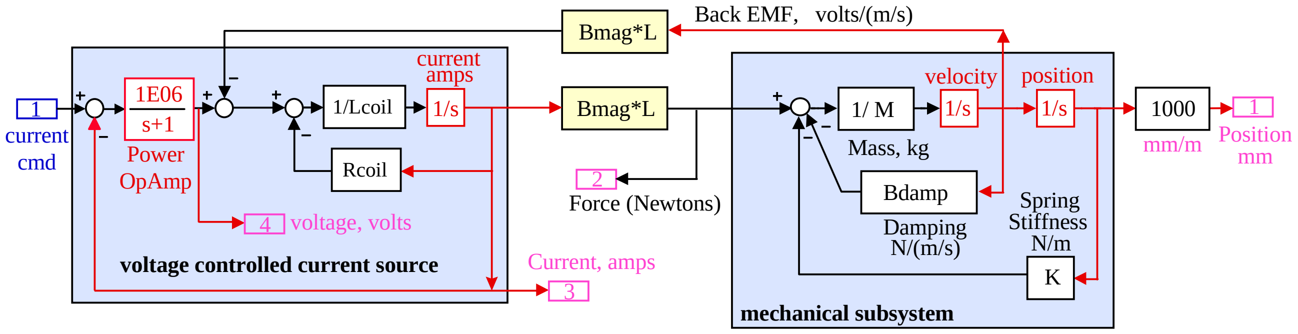
  Objective:
  - Understand Bode plots and their use for system analysis.
  - Understand phase and gain margins.
  - Recognize the importance of actuation limits

- Contents: 6 pages
  1 test questions

- Learning Time: 20 minutes

This module cover the analysis of a control system design. The system used in this example is comprised of a linear control system designed to control the position of a voice coil motor. Upon completion, you will be able to accomplish the objective listed here. Click the Forward arrow when you’re ready to continue.

Content Slide 1: Plant Model



[E2]
In a previous module we obtained a candidate control system for a model of an electro-mechanical system. The model consists of two coupled subsystems: a electrical subsystem comprised of a voltage controlled current source (VCCS) and a mechanical subsystem, consisting of a second order spring mass-damper system. The input to this system is a electric current command for the VCCS. The controlled output of this system is the position of the voice coil relative to a fixed reference frame. There are three uncontrolled system outputs: the force imposed on the voice coil, the electric current in the voice coil, and the electric voltage applied to the input of the voice coil. The force and current differ only by the scalar modeling constant: Bmag*L. To achieve the design specifications, a causal PID control structure was selected.

Content Slide 2: Causal PID control

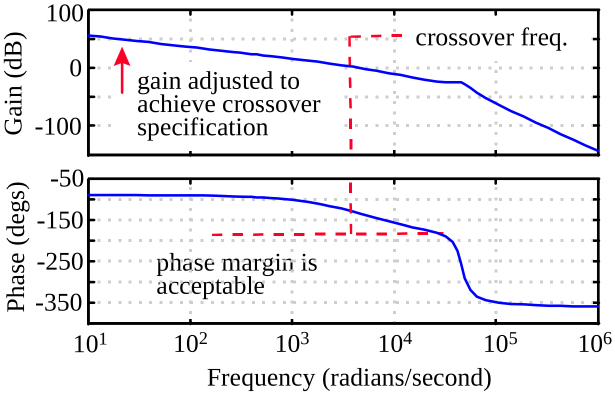
Causal PID control structure with differentiating filter to approximate a derivative calculation.

$$K(s) = k_p + \frac{k_i}{s} + \frac{k_d s}{(\tau s + 1)}$$

Closed Loop Transfer Function:

$$G_{closed-loop}(s) = \frac{G(s)K_{CPID}(s)}{[1 + G(s)K_{CPID}(s)]}$$

Frequency response of loop gain: $G(s)K_{CPID}(s)$



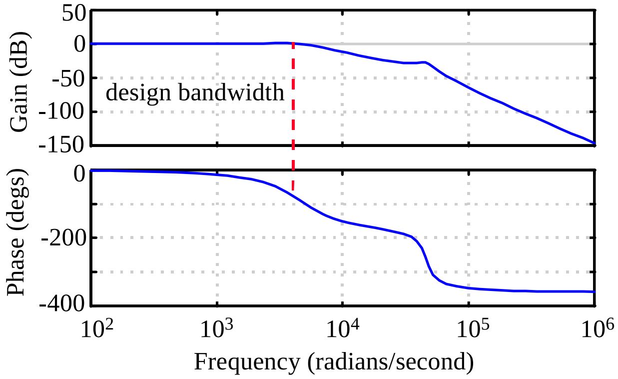
[E3 highlight left column]
A candidate design of a causal PID control system was performed in the previous. This controller is causal because we implemented the derivative term as a differentiating filter to approximate a derivative over the design bandwidth. The control structure consisted of 4 parameters: a proportional, integral and derivative gain, and a time constant used in the derivative approximation filter. When written as a single transfer function, K(s), this structure resulted in an s-domain pole at zero, a complex pair of zeros, and a overall scalar gain parameter.

[E3b highlight right column]
The control design was achieved using a loop shaping technique. Our design goal was to select a control structure such that the loop gain, G(s)K(s) resembled the function, “k/s”, over the design bandwidth. The desired bandwidth was then achieved by increasing the scalar controller gain such that the cross-over frequency was equal to the desired bandwidth. The cross-over frequency is define as that frequency corresponding to the point where the gain of G(s)K(s) is equal to 0 dB as shown in the figure above.

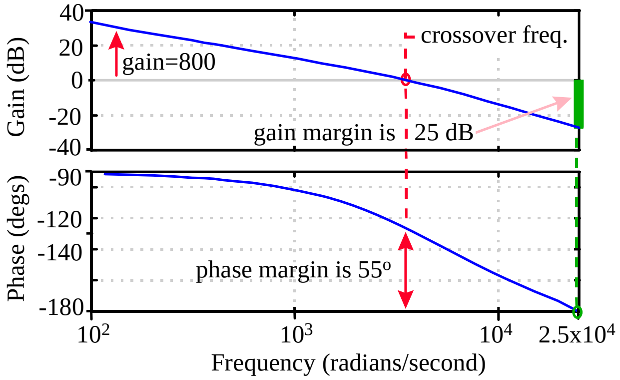
We have obtained what we believe is an acceptable design. In this module we have will analyze the design to evaluate its overall performance. As part of this analysis we will examine the other, uncontrolled, system outputs, specifically the force on the coil and the electric current in the coil.

Content Slide 3: Gain and Phase Margin

Closed loop frequency response



Loop gain plot of G(s)K(s) to obtain the gain and phase margin



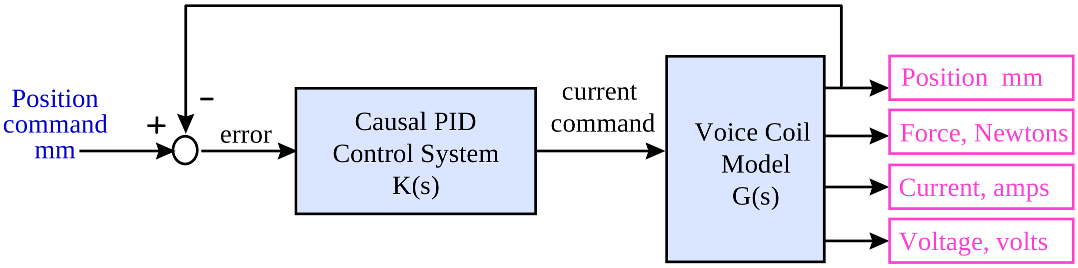
[E4]
With the controller pole, zero locations, and scalar gain selected in the previous module, we have a candidate controller. We can check the gain and phase margin of this proposed design.

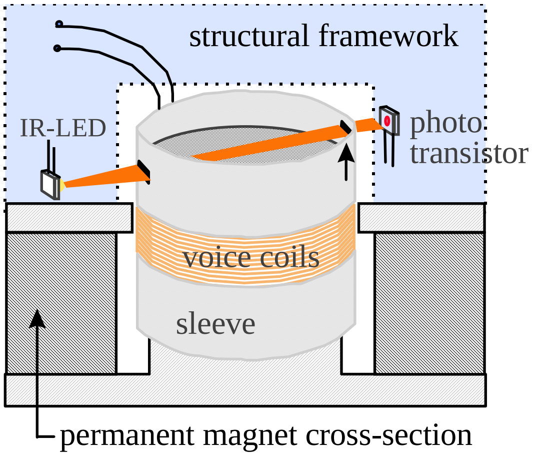
[E4 highlight right column]
The gain margin is the additional gain required for the gain plot to cross the 0 dB line at a frequency corresponding to a phase angle of -180 degrees. For the current system, the plot on the right shows that the gain margin is (-∞ and + 25 dB). This implies that the system gain would have to increase by a factor of 17.7, $(10^{25/20})$ before the system would go unstable. The controller gain can also drop to zero without the system becoming unstable, but this does not imply that performance is maintained.

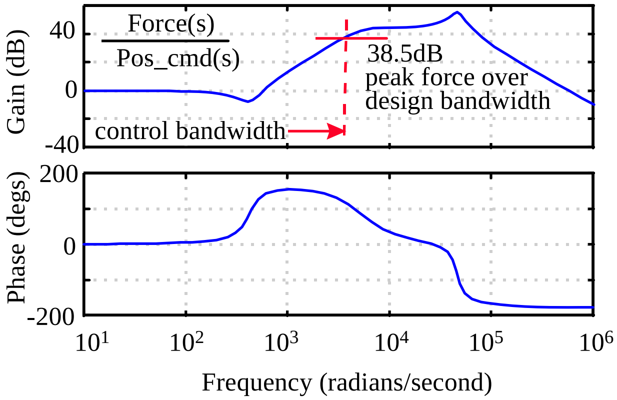
The phase margin is the additional phase required for the phase plot to reach -180 degrees at a frequency that corresponds to the gain value of 0 dB (the crossover frequency). For the current system, the gain margin is (-125)-(-180) = +55 degrees. At a frequency of 600 Hertz, this corresponds to a time lag of 0.25 millisecond, (55/360/600)

[E4 highlight left column]
The resulting closed loop frequency response, from commanded coil position to the measured voice coil position is shown in the figure on the lower left. Note that the gain plot is “flat” out to nearly 600 Hz (3770 rad/s). Our bandwidth specification has been satisfied.

Content Slide 4: Power Requirements


Position command mm
\+
–
error
Causal PID Control System K(s)
current command
Voice Coil Model G(s)
Position mm
Force, Newtons
Current, amps
Voltage, volts





$$\left.\frac{Force(s)}{Pos\_cmd(s)}\right|_{s\ =600\text{ Hz}} = 38.5\text{dB} = 84\text{N / mm}$$

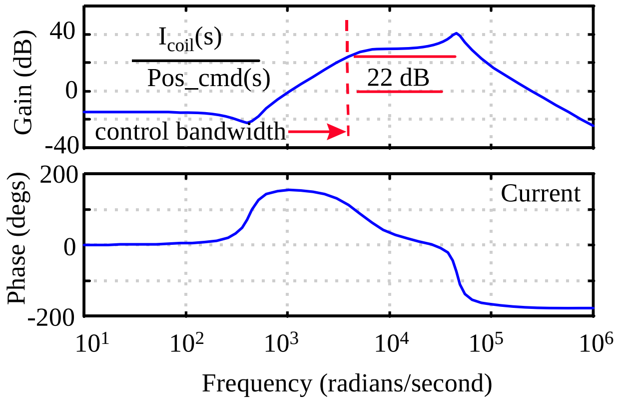
[E5 high light row 1]
We can examine the effects of the closed loop system on the other system outputs. Here we will consider the force exerted on the voice coil. The frequency response from commanded position in millimeters to the force in Newtons is shown in the Bode plot.

[E5 high light row 2]
The maximum force over the design bandwidth occurs at 600 Hertz and has a peak value of 84 Newtons for a command input magnitude of 1 millimeter. Note that there is a higher gain peak, but it occurs outside of the design bandwidth. We will assume that only sinusoidal inputs will be consider, so we only need to address the largest gain magnitude within the design bandwidth, which occurs at 600 Hz. Any voice coil force requires a reaction force on the frame holding the permanent magnetic. The point to be made here is that the framework has to be of sufficient stiffness such that it does not allow significant deflections. If the framework deflections become significant, then the vibration characteristics and mode shapes of the framework may have to be included in the model in order to design a proper control system.

Note that the framework houses the optical position sensor and this sensor is assumed to be fixed to the frame. If the framework undergoes deformation, the position sensor would measure this motion, but it would be the motion of the sensor within the frame, not motion of the voice coil relative to the frame. The assumption that there is one solid frame and that the position sensor is fixed to that frame would be violated. Thus our analysis here imposes a require upon the structure between the permanent magnet and the elements that make up the optical sensing system.

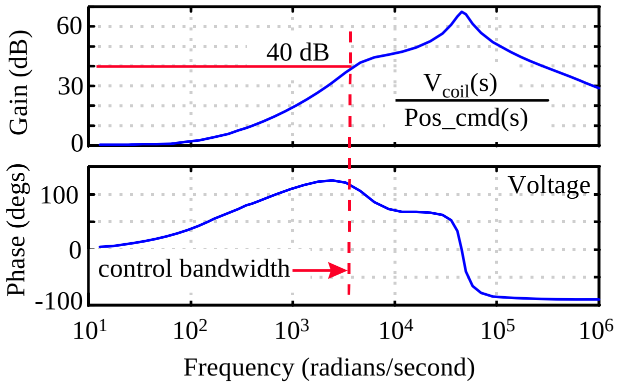
Content Slide 5: Power Requirements

Frequency response from position command to electric current delivered to voice coil.



At 600 Hz, current is $10^{\frac{22}{20}}$ = 12.5 amps.

Frequency response from position command to electrical voltage delivered to voice coil.



At 600 Hz, voltage is $10^{\frac{40}{20}}$ = 100 volts.

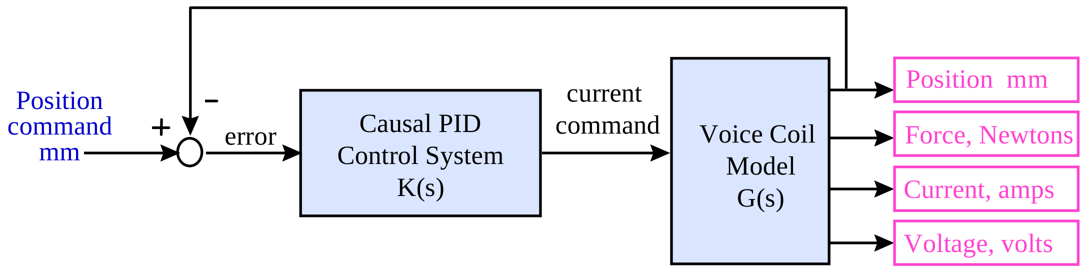
[E6]
One of the control design specifications called for a "reasonable" usage of power. Since we did not explicitly include the actuation requirements into the design (which we could do with a multivariable robust control design approach), we need to evaluate the power requirements. If our amplifier is not capable of delivering the required power, it will saturate or clip and our linear model assumption will be violated. Note that since we are only considering sinusoidal inputs, that we will only consider the frequency range from 0 to 600 Hertz and therefore not consider the peak gain outside of this region.

Consider the worst case sinusoidal command of ±1 millimeter at 600 Hz. At this point the amplifier is required to deliver a peak current of 12.5 amps. To achieve that current, the amplifier has to deliver a peak voltage of 100 volts. Whether or not this is a "reasonable" or feasible will depend on:

1) the amount of time operating at higher frequencies;
2) the ability of the system to dissipate the generated heat.
3) the current handling capabilities of the wire used in the voice coil.

Even at an RMS value of 8.8 amps the voice coil is not going to last long if it can not dissipate the generated heat. The coil resistance will generate about 400 watts of heat and the resistance is only going to increase with the addition of this heat. If the device has a short duty cycle it might tolerate this heat loading. This actuator is used in a valve to modulate air flow, so the air itself provide significant cooling. The point to be made here is that the application will determine what is deemed to be a reasonable amount of power.

Content Slide 6: Summary



Position
command
mm

+

–

error

Causal PID
Control System
K(s)

current
command

Voice Coil
Model
G(s)

Position mm

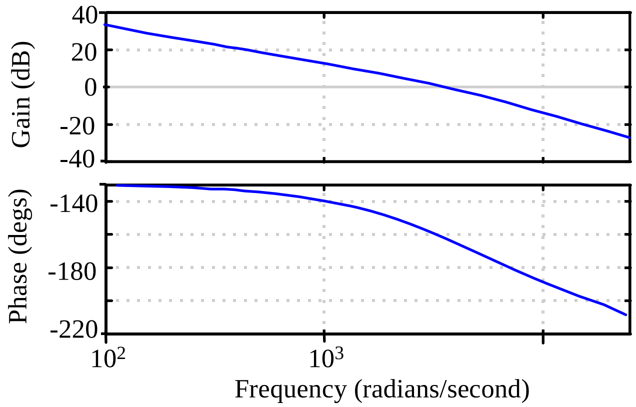
Force, Newtons

Current, amps

Voltage, volts

[E7]
We have demonstrated an approach to linear control system design on a single-input, single-output system. In this module we evaluated the control design by checking the gain and phase margins to see if they meet the design specifications. Next we investigated the effects of the control system on the other system outputs. We examined the reaction force generated in the housing to consider how it might affect the frame stiffness requirements. Finally, we examined the details of the power requirements. The power amplifier used in the voltage controlled current source must be capable of delivering the required power, otherwise we will have to consider the affects of clipping. Also, the self heating of the voice coil due to the peak electric current must also be considered in a thermal analysis of the system. By analyzing the system we have obtained information on how the control system affects uncontrolled system outputs and thus have taken a step towards a better design.

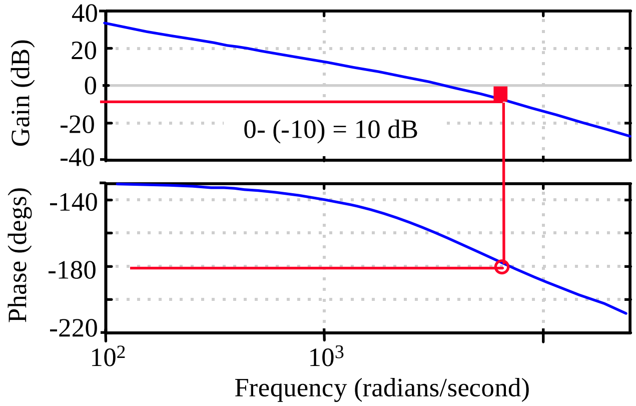
Content Slide 7: Question 1 Gain margin



The plot above is a plot of the loop gain G(s)K(s). What is the gain margin of this systems?

a) 40 dB
b) 30 dB
c) 20 dB
d) 10 dB CORRECT!
e) none of the above

Correct answer is (d), 10 dB



The gain margin is defined as zero minus the gain that corresponds to the frequency where the phase angle first equals (-180) degrees. In this example the frequency corresponding to the phase angle of (-180) degrees is about 6000 radians/second. The gain at this frequency is about -10 dB, so the gain margin is 0-(-10)=10dB.