An Integer Programming Formulation to Solve Lumosity's "Pet Detective"

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Abstract

I needed to get up to speed on optimization tools, so I used an easier problem as an aid to my learning process. The problem I chose to solve is the Pet Detective [1] game on Lumosity. I've played the game several times, but I reached a level that became difficult to solve quickly. I thought I could come up with a solver, (*not automated*), and learn about optimization tools in the process. I took an integer programming problem approach, using previously published formulations of the capacitated pickup-and-delivery, vehicle routing problem [2]. The purpose of this article is to share this experience as an educational tool. All files are available on GitHub [15]. An example problem and the formulation in AMPL and PuLP are included in the Appendix.

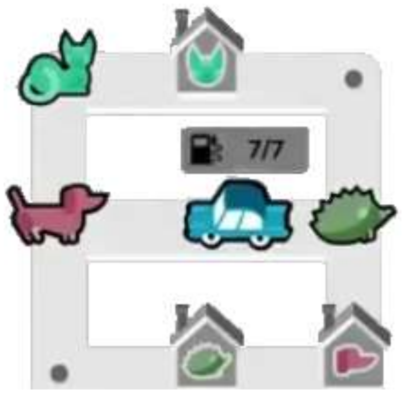
Keyword words and abbreviations

IP	Integer Programming	LP	Linear Programming
PDP	Pickup and Delivery Problem	VRP	Vehicle Routing Problem
CVRP	Capacitated Vehicle Routing Problem	TSP	Traveling Salesman Problem

Tools

There are a number of great tools for solving linear programming problems. I cycled through a number of them including LINGO [3], COIN OR, AMPL IDE, and RStudio [4], before settling in on SolverStudio [5,6] which is an add-in for Excel. Microsoft's Excel has its own built-in solver, but SolverStudio surpass that capability. SolverStudio supports several different modeling languages, (PuLP, COIN-OR, AMPL, GMPL, GAMS, Gurobi, CMPL, SimPy, and Python programming). SolverStudio also supports multiple solvers built into the various packages, including the web-based NEOS platform [7]. In the following, I use AMPL[8] and PuLP [9,10] as modeling languages. SolverStudio is a fantastic educational tool and greatly facilitated and accelerated my learning process.

The Pet Detective Problem

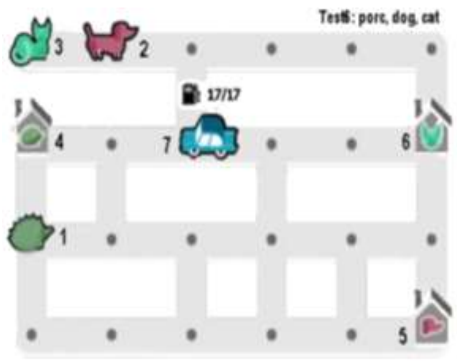


Pet Detective is a shortest route determination game. A vehicle is driven over a grid-like map to pick-up pets at various locations and to deliver those pets to their specific "homes" on the map. The vehicle is constrained to a maximum carrying capacity of four pets at a time, (which isn't relevant to the 3-pet problem shown on the left). The goal of the game is to figure out a route to deliver all of the pets with the shortest total distance. There is typically more than one optimal solution. The desired route length is provided as a goal to achieve, shown as "7" in the easy example game grid on the left.

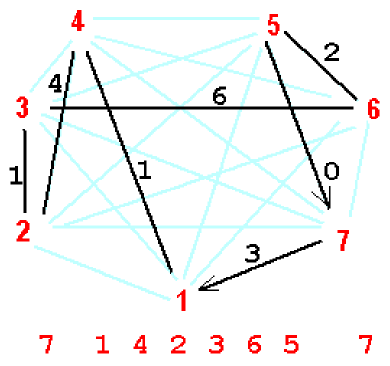
Formulation of the Optimization Problem

A “formulation” describes how the problem is modeled in order to present it to the “solver”. I followed the formulation in Lu & Dessouky’s 2002 paper [11], modifying it for a single vehicle and some other tweaks specific to this problem. The problem is viewed as a network with a set of nodes and arcs between nodes. This approach works but it leaves out information as discussed later in the conclusions. The formulation shown below follows the notation used in references [11,12].

A path through all of the nodes, called a “tour”, is considered to be a Hamiltonian cycle [13] where each node in the graph is visited just once. Subtour elimination (MTZ, DFJ) [14] is built-in to this formulation [11]. Multiple visits to a single node are avoided by defining “a visit” to a node to only occur only if either a pickup or delivery is made. Passing through a node without a drop-off or pickup is not considered to be “a visit”. The distance matrix (cost) is setup such that it only considers the shortest distance between nodes, which might be a path through a node with no pickup or drop-off.



In the example on the left, there are **n**=3 pickups, 3 drop-offs, and one depot for a total of 7 nodes. The route starts and ends at node 7, the depot. The nodes are shown in red text in the image on the right and the distances are in black text. The distance to return to the depot (5-7) is set to 0.



There are **n** pets or “customers”. Each customer has a pickup and delivery site. Denote N^{plus}, $\mathbf{N^+_r}$ = { 1, 2, …n} as the set of nodes making up the pickup requests and N^{minus}, $\mathbf{N^-_r}$ = { 1+n, 2+n, …2n} as the set of the delivery nodes (homes). Then the set of all service stops is N_r = union between N^+_r and N^-_r. There is only a single vehicle is this problem and it starts at the depot. We assign the depot to node number 2n+1. So the set of all of the nodes is indicated by the set “**N**”, (with no subscript). In the Pet Detective game, the game ends at the last home and there is no requirement to return to the starting location. In our formulation, the cycle requires a return to the depot, but the cost associated with the last move back to the depot is handled by assigning it a distance of zero. The vehicle capacity, denoted as **CAP**, is limited to 4 passengers. The problem is formulated as an integer programming problem as follows:

Decision Variables (solver output)

The decision variables are binary from the set {0, 1}.

$$x_{i,j} = \left\{ \begin{matrix} 1 & \mathit{if\ the\ vehicle\ travels\ from\ node\ i\ to\ node\ j} \\ 0 & \mathit{otherwise} \end{matrix} \right\}$$

$$b_{i,j} = \left\{ \begin{matrix} 1 & \mathit{if\ node\ i\ is\ before\ node\ j\ in\ the\ tour} \\ 0 & \mathit{otherwise} \end{matrix} \right\}$$

Objective Function (Cost)

$$\text{Minimize} \quad \textstyle\sum_{i \in N} \sum_{j \in N} \; c_{i,j}\, x_{i,j} \;\; ,$$

where

$$c_{i,j} = \; distance\ or\ cost\ associated\ with\ traveling\ from\ node\ i\ to\ node\ j.$$

> The cost matrix is required to be symmetric in this formulation. Since the solution is a Hamiltonian cycle, starting and ending at the depot, the costs between all of the drop-off nodes and the depot were forced to be zero, symmetrically. Thus the calculated objective matches the value in the puzzle. There is no cost to return to the depot.

Parameters

In the AMPL and PuLP models:

“Nodes” is used to describe the set of all nodes { N }
“NReduced” is used to describe the union { $N_r^+ \cup N_r^-$ }.
“Pickup” is used to describe the set { N_r^+ }.
“solve_result” is a text field to display the status of the solver output.
“Total_Cost” is the scalar value calculated from the objective function.

$$G_k = \begin{cases} +1 & if\ a\ passenger\ is\ picked\ up\ \ from\ node\ k \\ -1 & if\ a\ passenger\ is\ dropped\ off\ from\ node\ k \end{cases} \quad k \in \{ N_r^+ \cup N_r^- \}$$

Internal Variable

With zero initial passengers, the number of passengers in the vehicle when leaving node “j” is:

$$\text{zout}_{j} = G_j + \textstyle\sum_i^{2N} \{ b_{i,j} * G_i \}$$

This value is constrained to the capacity of the vehicle, but only needs to be tested for violation during a passenger pickup, not during dropoff. The variable **zout** is calculated and displayed as part of the solution, but it is calculated from **b**, so zout is not a “decision” variable.

Constraints (subject to)

In the following, I’m using the equation numbering from reference [11], but I skip the equation numbers that are not being used.

Basic network constraints

One entry and one exit per node. This will appear in the solution “**x**” matrix as one nonzero entry per row and one nonzero entry per column, as shown in the example solution in the Appendix.

$$\sum_{j \in N} x_{i,j} = 1, for\ all\ i \in N \qquad \text{for all } i \in N.\ \text{Summation of the columns per each row} \qquad (1)$$

$$\sum_{i \in N} x_{i,j} = 1, for\ all\ j \in N \qquad \text{for all } j \in N.\ \ \text{Summation of the rows per each column} \qquad (2)$$

Copy constraints (*get the subscripts right.   A/B means contained in set A, but not in the set B*)

$b_{k,i} \leq b_{k,j} + (1 - x_{i,j})$ for all arc(i,j) ∈ N /{arc(2n+1) and k∈N / {i} (3)

$b_{k,j} \leq b_{k,i} + (1 - x_{i,j})$ for all arc(i,j) ∈ N /{arc(2n+1) and k∈N / {i} (4)

$x_{i,j} \leq b_{i,j}$ for all arc(i,j) ∈ N (5)

The copy constraints, equations 3&4, are such that if node i is immediately before node j in the tour, ($x_{i,j}$=1), then these equations force $b_{k,i} = b_{k,j}$ for all k∈ N and k≠i. Equation 5 makes sure that b(i,j) is set if x(i,j) is set and if b(i,j) is zero, then x(i,j) is also zero.

Diagonals: The matrix diagonal terms are all zero for both **x** and **b**. EQ 5 above enforces this for **x**.

$b_{i,i} = 0$ for all i ∈ N (6)

Priors Constraints

In the tour, a drop-off node will never occur before the corresponding pickup node (equation 7) and a pickup node always occurs before the corresponding drop-off node, (equation 8). These cells are shown as blue shaded diagonal cells in the b matrix for the solution shown in the Appendix. Equation (5) combined with equation (7) results in the blue shaded diagonal cells in the x matrix for the solution shown in the Appendix.

$b_{n+i,\, i} = 0$ for all i ∈ N_r^+ (7)

$b_{i,\, n+i} = 1$ for all i ∈ N_r^+ (8)

Capacity Constraint

Since we only have one vehicle in the Pet Detective problem, we skip the pairing constraint in equation (9) and jump to equation (10) for the vehicle carrying capacity. I've already mention the internal variable zout $_j$, which is the equal to the number of passengers/pets in the vehicle upon departing node j. Note the use of the set N_r^+, since the capacity only needs to be tested when a customer is picked up.

$\text{zout}_j = G_j + \sum_i^{2N}\{b_{i,j} * G_i\} \leq m$ for all i ∈ N_r^+ where m = 4 in Pet Detective. (10)

First/Start Constraints (departing the depot)

This constraint says that the first pickup won't be from a drop-off node. Note the use of the set $N_r^{\mp}$. The second equation is redundant with equation (5).

$b_{2n+1,i} = 1$ Start for all i ∈ N_r^+, last row of b, values = 1 (11)

$x_{2n+1,n+i} = 0$ First for all i ∈ N_r^+, last row of x

Last/End Constraints (returning to the depot)

This constraint indicates that the last move, returning back to the depot, will not be from a pickup node. Note the use of the set N_r^+. The second equation is redundant with equation 5 and is probably is removed during pre-solve.

$b_{i,2n+1} = 1$ End for all i ∈ N_r^+ last column of b, values = 1 (14)

$x_{i,2n+1} = 0$ Last for all i ∈ N_r^+ last column of x

We differ from reference [11] on equations (11) and (14) because we are only using a single node for both the first and last node. The last row/column of the b-matrix may be unnecessary, since it is pre-known with this formulation.

Constraints for Speed Improvements

Reference [11] included a number of constraints to increase the solver speed. I attempted all but the Transfer Prior constraint, which was complicated to implement. I had success with all that I tried independently, but when combined some of the constraints were no longer of value, so I ended up using a subset and commenting out the rest. Only those that are used are listed below.

Valid Equalities - Vehicle Return Constraint

Because Pet Detective only has one vehicle and because I chose to only have one node for both the starting and end location, this constraint was reduced to making the last column in the b-matrix all ones, except for the diagonal term. It would probably be easier to code the values as one directly, instead of using the summation below, but the summation did improve the solution speed.

$$\sum_{k=1}^{2n+1} b_{k,2n+1} = 2n \qquad \text{for all i} \in \{ N_r^+ \cup N_r^- \} \tag{25}$$

Valid Equalities – B Equality Constraint

This constraint is from the definition of the b-variables and did result in a significant speed improvement, (on the order of 10% reduction in the solution time).

$$b_{i,j} + b_{j,i} = 1 \qquad \text{for all i} \in \{ N_r^+ \cup N_r^- \} \tag{26}$$

Known Objective Constraint

Pet Detective provides the target objective, so there is no reason to search beyond that value. Most solvers will search until they concluded that an optimal has been reached, but this takes time for the solver to come to this conclusion. The following provided a small decrease in the solution time by avoiding search beyond the target objective. I noticed about a 10% reduction in the solution time using this constraint.

$$\sum_{i \in N} \sum_{j \in N} c_{i,j}\, x_{i,j} >= \text{KNOWN\_OBJ}$$

The Solution

The AMPL NEOS server and the PuLP + Coin-OR Branch and Cut (CBC) solver worked well, but there are a lot of options. I experimented with a few of these with no conclusive improvement, except for using an initial feasible solution. PuLP does not have a direct way to command an initial feasible solution [16] (i.e. “mipstart”). As an alternative, I got in the habit testing using SolverStudio+PuLP to generate the CBC model file, which I would then “*import”* into the command-line version of CBC. This allowed me to set “*verbose 1*” to get more feedback and to use “mipstart” to define an initial feasible condition. Note that PulP generates a “.lp” and a ‘.mps” file, but the “.mps” file is transient and it is removed after the solver has completed, so it is difficult to capture on small problems.

You can then run “cbc” from the command line or interactively to monitor the solution process. I used the minimize option: “min”, “verbose 1” and “solve” to generate the solution. Note that you can “import” the “**.lp**” file in “cbc” issue the “solve” command and then save the results to file using the “solu” command.

This CVRP formulation of Pet Detective *works*, but it is not particularly fast. Table 1 shows the solution times using the AMPL model using the NEOS server for a set of puzzles from 3 to 11 passengers. The larger problems with 10 and 11 pets took in excess of 2 hours to solve, (*hardly interactive*). Note that Pet10 was harder to solve than Pet11.

Table 1 AMPL Performance Data (CAP=4)

# Pets (n)	# of Nodes (2n+1)	Time (sec)	adjusted binary variables	constraints				
				# linear	# non-zeros	# equality	# inequality	# non-zero
3	7	0.03	57	255	696	38	217	30
4	9	0.24	112	698	1978	66	632	56
5	11	0.16	180	1467	4120	102	1365	90
6	13	1	258	2673	7788	146	2527	132
7	15	300	357	4405	12726	198	4207	182
8	17	4425	472	6763	19888	258	6505	240
9	19	3632	603	9840	29016	326	9514	306
10	21	13145	750	13733	40580	402	13331	380
11	23	10700	913	18538	54868	486	18052	462

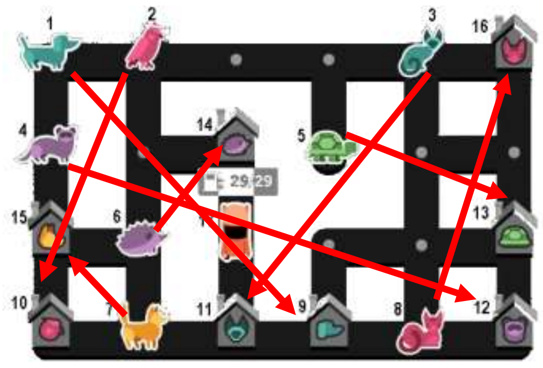
The solution times for the PuLP formulation, using a Dell Precision T5500 workstation running under 64-bit Windows 10 with Intel Xeon processors and 24 Megs of ram, are shown in Table 2. When using the PuLP “solve” command in SolverStudio, PuLP uses **cbc** as the solver. Running the same .lp model from the command line version of **cbc** provides some extra information for comparing the size of models. I’ve just scratched the surface of **cbc**.

Table 2 PuLP Formulation Solved using CBC (64-bit) on PC/Windows10

# Pets (n)	# of Nodes (2n+1)	Solver Studio Time (sec)	CBC cmd line Time (sec)	CBC Enum Nodes	CBC cmd line Iterations	Comment: Execution in SolverStudio and using the command line version of CBC
3	7	0.7	-			Run in Excel using SolverStudio+PuLP
4	9	1.1	-			“
5	11	1.5	-			“
6	13	2.4	-			“
7	15	8.6	9	1559	330761	Both SolverStudio and CBC
8	17	269	267	4901	1263020	Both SolverStudio and CBC
9	19	4808	9365	47199	16067299	Both SolverStudio and CBC
10	21		24133	56483	32975824	Run from CBC Command line
11	23	2423	2939	1663	1799546	Both SolverStudio and CBC

Conclusions

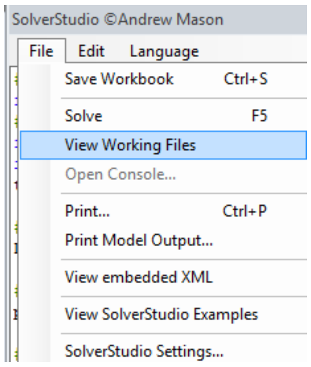
In most instances, a human is able to solve the puzzle faster than this formulation, (particularly if you consider the manual generation of the distance matrix). I believe this is due to the fact that this formulation leaves out details about the grid and just looks at the puzzle as a network. You don't get any information about overlaying routes for multiple passengers with this formulation. I know that when I solve the puzzle manually, I estimate a solution moment as to whether the main flow of the solution needs to be clockwise or counter clockwise and I look for redundant paths. For example, "2-1-4" and 10-9-12 are patterns that fit the problem on the left. This information about the grid and overlaying paths is not visible in this formulation. None-the-less, it is interesting to see the linear programming solver achieve a valid solution.



There may be better formulations for this particular problem, but this project has served its purpose and allowed me to get up to speed on optimization tools like AMPL, PuLP, and SolverStudio.

SolverStudio

I love SolverStudio. It's a great tool and greatly facilitated my learning process. I was able to figure out most things by looking at the provided examples and searching the internet, including modifying the PuLP source to save a log file [17]. When I didn't get enough debug information from SolverStudio, I would just jump to the AMPL IDE to run the model. Similarly, I would run the PuLP model in SolverStudio with the command "solve" commented out and then go to "Working Files" folder to extract the raw model file which I could them import into the command line version of CBC. Also, being able to switch to the NEOS server for large problems is a huge benefit.



The only drawback that I see to SolverStudio, is that unlike most other environments (AMPL & LINGO), it combines the data and model into a single Excel .xslx file, which results in duplicate models for various sets of data and as a non-text file, it does not lend itself to source differencing for version control. I don't think I would use SolverStudio for production work without addressing this particular shortcoming, but as an education tool, it is hard to beat.

Automating the Process

My reason for learning SolverStudio was to get up to speed on optimization tools for another project. Whereas I could have generated an image parser to automate the generation of the distance/cost matrix from the Pet Detective grid, I chose not to spend my time on this task. As such, the most tedious part of this "solver" is the manual generation of the distance/cost matrix. Errors in this matrix, particularly those that make the matrix asymmetric, will vastly increase the runtime. Long runtimes are an indication of a user typo error.

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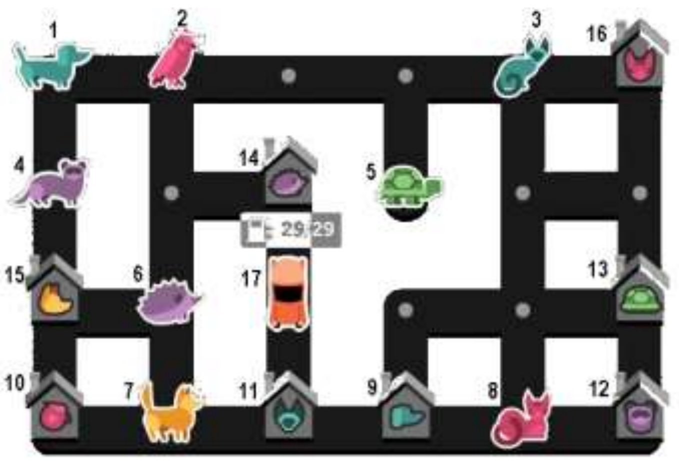
Files

All of the Excel source files for this investigation are available on GitHub [15].

Appendix: Example Problem with Cost/Distance Matrix (PetD8)

PetD8 contains 8 pets to be picked up and delivered to 8 homes for a total of 16 service nodes and 17 nodes total, including the depot. The pet nodes are numbered first, 1-8, and the homes are numbered 9-16, with the "depot" or starting/end point labeled 17. The vehicle capacity (CAP) is 4 pets. The goal objective is 29. The pet nodes (pickup) have a value G=+1, whereas the home nodes (drop-off) have G=(-1). One optimal solution is the node numbered order beginning and ending with "17", shown to the right.



17: 7 6 14 2 1 4 15 10 9 8 5 3 16 13 12 11 :17

N	17			M	8			KNOWN_OBJ		29			**CAP**	4			
Nodes	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	**17**
NReduced	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	
Pickup	1	2	3	4	5	6	7	8									

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
G	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1
zout																

cost	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	0	1	4	1	4	3	4	7	6	3	5	8	7	3	2	5	4
2	1	0	3	2	3	2	3	6	5	4	4	7	6	2	3	4	3
3	4	3	0	5	2	5	6	3	4	7	5	4	3	5	6	1	6
4	1	2	5	0	5	2	3	6	5	2	4	7	8	4	1	6	5
5	4	3	2	5	0	5	6	5	6	7	7	6	5	5	6	3	6
6	3	2	5	2	5	0	1	4	3	2	2	5	6	2	1	6	3
7	4	3	6	3	6	1	0	3	2	1	1	4	5	3	2	7	2
8	7	6	3	6	5	4	3	0	1	4	2	1	2	4	5	4	3
9	6	5	4	5	6	3	2	1	0	3	1	2	3	3	4	5	0
10	3	4	7	2	7	2	1	4	3	0	2	5	6	4	1	8	0
11	5	4	5	4	7	2	1	2	1	2	0	3	4	2	3	6	0
12	8	7	4	7	6	5	4	1	2	5	3	0	1	5	6	3	0
13	7	6	3	8	5	6	5	2	3	6	4	1	0	6	7	2	0
14	3	2	5	4	5	2	3	4	3	4	2	5	6	0	3	6	0
15	2	3	6	1	6	1	2	5	4	1	3	6	7	3	0	7	0
16	5	4	1	6	3	6	7	4	5	8	6	3	2	6	7	0	0
17	4	3	6	5	6	3	2	3	0	0	0	0	0	0	0	0	0

Note the zero values in the violet squares force the return to depot cost to zero while keeping the cost matrix symmetric.

The solution to the PET8 problem is presented below. The size of the matrix provides an idea of the number of binary variables for which a solution must be obtained. Some of the constrained cells are also shown colored with a blue background. All of the values in b and x are binary.

b	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	0	0	1	1	1	0	0	1	1	1	1	1	1	0	1	1	1
2	1	0	1	1	1	0	0	1	1	1	1	1	1	0	1	1	1
3	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	1	1
4	0	0	1	0	1	0	0	1	1	1	1	1	1	0	1	1	1
5	0	0	1	0	0	0	0	0	0	0	1	1	1	0	0	1	1
6	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1
7	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1
8	0	0	1	0	1	0	0	0	0	0	1	1	1	0	0	1	1
9	0	0	1	0	1	0	0	1	0	0	1	1	1	0	0	1	1
10	0	0	1	0	1	0	0	1	1	0	1	1	1	0	0	1	1
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
12	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1
13	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	1
14	1	1	1	1	1	0	0	1	1	1	1	1	1	0	1	1	1
15	0	0	1	0	1	0	0	1	1	1	1	1	1	0	0	1	1
16	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	1
17	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0

x	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
5	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
7	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
12	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
14	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
17	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0

solve_result	Total_Cost
solved	29

Order

17 7 6 14 2 1 4 15 10 9 8 5 3 16 13 12 11 17

Appendix: AMPL Model Equations

```
set Nodes ordered;                  # All Nodes
set Nreduced ordered;               # Nodes{1:2M}
set Pickup ordered;                 # Nodes{1:M}

param N integer;                    # all node count
param M integer;                    # pickup count
param CAP integer;
param KNOWN_OBJ integer;
param G{Nreduced} integer;
param zout{Nreduced} integer;
param cost {Nodes,Nodes} >= 0;      # Costs


# Decision Variables -------------------------------------------------------
# x contains the (0,1) binary variable to define the arcs used
var x {i in Nodes, j in Nodes} binary ;
# b contains the (0,1) binary variable to define if node i occurs before j
var b {i in Nodes, j in Nodes} binary ;
# zout is not a decision variable. It’s calculated from the resulting G[j],B[i,j]

# OBJECTIVE -----------------------------------------------------------------
# The objective function is added
minimize Total_Cost:
   sum {i in Nodes, j in Nodes} cost[i,j] * x[i,j];


# CONSTRAINTS ---------------------------------------------------------------
#1 Basic Network, one entry, one exit per node
subject to LinksOutPerNode{k in Nodes}:
    sum {j in Nodes} x[j,k] = 1;
#2
subject to LinksIn_PerNode{k in Nodes}:
    sum {i in Nodes} x[k,i] = 1;

#Copy Constraints.  If x[i,j]=1, then node i isimmediately before node j,
#3                  and b[k,i]=b[k,j] for all k, where k<>i
#                   Does not include DEPOT
subject to copy1{i in Nreduced, j in Nreduced, k in Nreduced: k<>i }:
    b[k,i] <= b[k,j] + (1-x[i,j]);
#4
subject to copy2{i in Nreduced, j in Nreduced, k in Nreduced: k<>i and k<>j }:
    b[k,j] <= b[k,i] + (1-x[i,j]);
#5
subject to copy3{i in Nreduced, j in Nreduced: i<>j} :
    x[i,j] <= b[i,j];
```

```
#6 Diagonals
subject to DiagonalsB{i in Nodes}:
    b[i,i] = 0;
#not useful
subject to DiagonalsX{i in Nodes}:
    x[i,i] = 0;

#7 PRIOR CONSTRAINTS, pickup before delivery
subject to Prior1{i in Pickup}:
    b[M+i,i] = 0;
#8 PRIOR Pickup must be before delivery
subject to Prior2{i in Pickup}:
    b[i,M+i] = 1;

#10 CAPACITY CONSTRAINT tested at pickup location only
subject to Capacity{j in Pickup}:
   sum {i in Nreduced} b[i,j]*G[i] <= CAP-G[j];

#11 FIRST node won't be a drop-off
subject to First{i in Pickup}:
    x[N,M+i] == 0;
#14 LAST node won't be from a pickup
subject to  Last{i in Pickup}:
    x[i,N] == 0;

#Extra since Depot is start and end.
Subject to Start{i in Nreduced}:
    b[N,i] == 1;
subject to End{i in Nreduced}:
    b[i,N] == 1;


#Lu&Dessouky SPEED IMPROVEMENT CONSTRAINTS
#4 (eq25) Vehicle Return Constraint cuts some fractional solutions (1 vehicle,
simplified)
subject to VehicleReturn:
    sum {k in Nreduced} b[k,N] == N-1;

#5 (eq26) Equality Constraint from definition of the b variables
subject to Equality26{i in Nreduced, j in Nreduced: i<>j}:
    b[i,j] + b[j,i] == 1;

#Limit Objective to Known Target Objective
subject to MIN_OBJ:
    sum {i in Nodes, j in Nodes} cost[i,j] * x[i,j] >= KNOWN_OBJ;
```

```
#==================================================================
# Process Results
#==================================================================
# Get the data from the sheet
data SheetData.dat;
option solver cbc;       #switch to COIN-CMDC solver
option show_stats 1;
#option gentimes 1;      # display summary of resources used
#CPLEX OPTIONS FOR AMPL
#option mipdisplay 3;            #frequency of displaying branch-bound info
#option lowerobj KNOWN_OBJ+1;    #
#option tunedisplay 3;   # exhaustive printing
solve;

#solution - pull the SolverStudio variables on to the sheet
display x > Sheet;
display b > Sheet;

#solver info
display solve_result > Sheet;
display Total_Cost > Sheet;

#Calculate zout
let {j in Nreduced} zout[j] := G[j] + sum{i in Nreduced}round(b[i,j],0)*G[i];
display zout > Sheet;

#EXTRA DEBUG INFO
display  _ampl_elapsed_time,  _ampl_system_time,  _ampl_user_time;
display _solve_elapsed_time, _solve_system_time, _solve_user_time;


                             *** END OF AMPL MODEL ***
```

Appendix: PuLP Model Equations

```
# Import PuLP modeler functions
from pulp import *
# Import math functions
from math import *
import datetime as dt
t1 = dt.datetime.now()

# Parameters
Debug = 0

# Create the 'prob' variable to hold the data
prob = LpProblem("PetProject", LpMinimize)

# Creates a list of tuples containing all the possible Nodes, skip i=j
ARCS = [(i,j) for i in Nodes for j in Nodes if i!=j ]

# Decision Variables ---------------------------------------------------------
# x contains the (0,1) binary variable to define the arcs used
x = LpVariable.dicts("x",(Nodes,Nodes),0,1,LpBinary)
# b contains the (0,1) binary variable to define if node i occurs before j
b = LpVariable.dicts("b",(Nodes,Nodes),0,1,LpBinary)
# z is not a decision variable. It's calculated from the resulting G[j],B[i,j]

# OBJECTIVE ------------------------------------------------------------------
# The objective function is added to 'prob' first
prob += lpSum([x[i][j]*costs[i,j] for (i,j) in ARCS ]),"Total_Costs"
#print prob


# CONSTRAINTS ----------------------------------------------------------------
#1 Basic Network, one entry, one exit per node
for k in Nodes:
    prob += lpSum([x[j][k] for j in Nodes]) == 1,"Basic_One_Entry_%d"%k  #sumcols
#2
for k in Nodes:
    prob += lpSum([x[k][i] for i in Nodes]) == 1,"Basic_One_Exit_%d"%k


#Copy Constraints.  if x[i,j]=1, then node i isimmediately before node j,
#3                  and b[k,i]=b[k,j] for all k, where k<>i
for i in NReduced:
    for j in NReduced:
        for k in NReduced:
            if k != i:
                prob += b[k][i] <= b[k][j] + (1-x[i][j]),"copy1_%d_%d_%d"%(i,j,k)
```

```
#4
for i in NReduced:
    for j in NReduced:
        for k in NReduced:
            if k != i:
                prob += b[k][j] <= b[k][i] + (1-x[i][j]),"copy2_%d_%d_%d"%(i,j,k)
#5
for i in Nodes:
    for j in Nodes:
        prob += (x[i][j] <= b[i][j] ),"copy3_%d_%d"% (i,j)

#6 Diagonals
for i in Nodes:
    prob += b[i][i] == 0, "DiagonalB_%d"%i
#for i in Nodes:   #(not useful)
    prob += x[i][i] == 0, "DiagonalX_%d"%i

#7 PRIOR CONSTRAINTS, pickup before delivery
for i in Pickup:
    prob += b[M+i][i] == 0,"Prior1_%d"%i
#8 PRIOR Pickup must be before dlivery
for i in Pickup:
    prob += b[i][M+i] == 1,"Prior2_%d"%i

#10 CAPACITY CONSTRAINT with vehicle initially having zero passengers
for i in NReduced:
    prob +=  lpSum(b[j][i]*G[j] for j in NReduced) <= CAP-G[i],"Capacity_%d_%d"%
(i,CAP-G[i])


#11 FIRST node won't be a drop-off
for i in Pickup:
    prob += x[N][M+i] == 0,"First_%d"%i

#14 LAST node won't be from a pickup
for i in Pickup:
    prob += x[i][N] == 0,"Last_%d"%i

#Extra since Depot is start and end.
for i in NReduced:
    prob += b[N][i] == 1,"Start_%d"%i
for i in NReduced:
    prob += b[i][N] == 1,"End_%d"%i

#Lu&Dessouky SPEED IMPROVEMENT CONSTRAINTS
#1 (eq 21) Transfer Prior Constraint
#  (too hard to implement because of arbitrary collection)
```

```
#2 (eq 22,23) Adjacent Prior Constraints DOESN'T HELP
#for k in Nodes:
#    for i in Pickup:
#        if(i != k and i+M != k):
#           prob += b[k][i]+b[k][i+M] >= x[i  ][k]+x[k][i], "AdjacentPriorA_%d_%d"%(k,i)
#            prob += b[i][k]+b[i+M][k] >=
                                x[i+M][k]+x[k][i+M],"AdjacentPriorB_%d_%d"%(k,i)

#3 (eq24) Pairing Prior Constraint DOESN'T HELP
#for i in Pickup:
#    prob += lpSum(b[k][i] for k in Nodes)+1 <=
                         lpSum(b[k][i+M] for k in Nodes),"PairingPrior_%d"%i

#4 (eq25) Vehicle Return Constraint cuts some fractional solutions (1 vehicle)
prob += lpSum([b[k][N] for k in NReduced]) == N-1,"VehicleReturn"

#5 (eq26) Equality Constraint from definition of the b variables
for i in NReduced:
    for j in NReduced:
        if i != j:
            prob += b[i][j] + b[j][i] == 1,"Equality26_%d_%d"%(i,j)

#Limit Objective to Target Objective or above to truncate minimum search
prob += prob.objective >= KNOWN_OBJ, "MIN_OBJ"


#SOLVER ------------------------------------------------------------------
prob.writeLP("PD9.lp")              # Write the problem as an LP file
prob.solve()                        # Solve the problem using the default solver
#prob.solve(COIN_CMD(msg=1))        #solve using CBC with logging
#prob.solve(COIN_CMD(msg=1, fracGap = 0.0))          #solve using CBC



#CALC TIME  --------------------------------------------------------------
t2 = dt.datetime.now()
dt = t2-t1
print 'Time: %6.3f seconds' %(dt.seconds+dt.microseconds/1e6)  #modulo printing


#DISPLAY SOLUTION  -------------------------------------------------------
solve_result = LpStatus[prob.status]
''' write to spreadsheet '''
solve_result = LpStatus[prob.status]
for (i,j) in ARCS:
    xout[i,j] = x[i][j].varValue
for (i,j) in ARCS:
    bout[i,j] = b[i][j].varValue
```

```
print("Status:", LpStatus[prob.status])
TotalCost = lpSum([x[i][j].varValue * costs[i,j] for (i,j) in ARCS ])
print 'TotalCost = %s' % TotalCost
Total_Cost = value(prob.objective)

#display solution X
print ' x  ',
for i in Nodes:
    print '%2.0f' % i,
print
for i in Nodes:
    print '%2.0f  ' % i,
    for j in Nodes:
        if i!=j:
            print  '%2.0f' % x[i][j].varValue ,
        else:
            print  " 0",
    print
print

#Diplay Capacity at nodes
print ' z '
for i in NReduced:
    z = G[i]
    for j in NReduced:
        z = z + G[j]*b[j][i].varValue
    print '%2.0f'%z,
    zout[i] = z          #write to spreadsheet
print
```

*** End of PuLP Model ***

* * * EOF * * *